SPECTROGRAPHIC INVESTIGATIONS OF THE STRUCTURE OF JUPITER'S DECAMETRIC RADIO EMISSION SOURCES

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Abstract

A possibility to determine the structure of distributed sources of Jupiter's decametric radio emission using spectrographic studying of statistical characteristics of fluctuating planetary emission is analyzed. Observational conditions are determined when measurements of frequency spectra and frequency–temporal cross correlation function of intensity fluctuations can be used to determine spatial structure of Jupiter's decametric radio emission sources. Theoretical results are compared with known results of experimental investigations of statistical characteristics of Jupiter's decametric emission. Frequency non–stationarity of intensity fluctuations of emission received by spectrograph in a wide frequency band is emphasized. In particular, it is found that narrow band measurements of frequency–temporal cross correlation of intensity fluctuations of decametric radio emission could be used as effective diagnostic tool for determining weak spatial gradients of Jupiter's magnetic field.

1 Introduction

Spectrographic studies of Jupiter's decametric radio emission have been carried out for many years [see, for example, Lecacheux et al., 1998a; Queinnec and Zarka, 1998, and references therein]. In particular, investigations of interplanetary scintillation frequency drifts of Jovian decametric emission (DAM) were used to verify a hypothesis on spatial distribution of DAM sources along Jupiter's field lines [Genova and Boischot, 1981; Boischot et al., 1987]. (Later it was confirmed by a two–frequency measurements that the decametric planetary radio emission fluctuated due to scattering off interplanetary plasma irregularities.) However, it should be mentioned that mainly quasi–regular characteristics of radio emission such as slopes of frequency drifts in dynamic spectra were investigated in

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Genova and Boischot [1981] and Boischot et al. [1987]. Naturally, it limited quantitative possibilities of the technique proposed in Genova and Boischot [1981]. At the same time it is interesting to consider a possibility to extract some information on Jovian decametric radio emission sources from spectrographic investigations of statistical characteristics of fluctuating planetary emission.

2 Spectral characteristics of Jupiter's decametric radio emission

The main feature of spectrographic investigations of DAM is their wide frequency band: usually emission covers from several to several ten MHz [Lecacheux et al., 1998a; Queinnec and Zarka, 1998]. Frequency non–stationarity is a distinctive feature in statistical characteristics investigations of fluctuating emission received by spectrograph.

Let us consider the following modeled problem. Distributed source of emission with plane aperture is situated in $z = const$ plane. There is a scattering phase screen at distance z_1 from it. Emission from the source passes through the screen and is received at observational point at distance z_2 from the screen by a spectrograph that uses linear frequency sweeping $f(t) = f_o(1 + \alpha t)$, where f_o is the initial frequency, and $\alpha = (1/f_o)(\mathrm{d}f/\mathrm{d}t)$ is the normalized sweeping rate. The screen causes random phase fluctuations of an emission $s(\overrightarrow{p}, f, t)$ passing through it, where $\overrightarrow{p}(x, y)$ is the radius vector in the screen plane $z = 0$. The source is supposed to be a continuous aggregate of point sources spatially and simultaneously frequency spread.

Following [Rytov et al., 1978] we can describe the wave field at the observational point from a separate source with emission frequency $f(t)$ by Fresnel approximation

$$\begin{aligned} E(0, z_2) = \; & -i\frac{k\mathrm{e}^{ikR_0}(z_1+z_2)^3}{2\pi z_1 z_2 R_0^3} \int\int_{-\infty}^{\infty} \mathrm{d}\overrightarrow{\rho} \exp\left[is\left(\overrightarrow{\rho} + \frac{z_2}{z_1+z_2}\overrightarrow{\rho_s}, f, t\right)\right] \\ & \exp\left\{i\frac{k}{2L}\left[\overrightarrow{\rho}^2 - \frac{(\overrightarrow{\rho}\overrightarrow{\rho_s})^2}{4R_0^2}\right]\right\} \end{aligned} \tag{1}$$

Here $L = z_1 z_2 R_0/(z_1 + z_2)^2$, $R_0 = \sqrt{(z_1 + z_2)^2 + \rho_s^2}$ is the distance from the source to the observational point, $\overrightarrow{\rho_s}$ – radius vector in the source aperture plane, $k = 2\pi f/c$ – wave number (c is vacuum light velocity).

We will consider further "freezing" of random screen inhomogeneities drifting in the screen plane $z = 0$ with velocity $\overrightarrow{V}_H$, and will analyze the most typical case of short radio wave diffraction in interplanetary plasma, namely, the case of strong phase fluctuations on the screen (mean square of phase fluctuations $\bar{s}^2 \gg 1$). Then, using (1) and following [Rytov et al., 1978; Gershman et al., 1984] after some cumbersome but not complex calculations the next expression for mean spectrum of intensity fluctuations of received by spectrograph emission in frequency band ΔF can be derived

$$\begin{aligned} \widetilde{W}_{\Delta I} = \; & \frac{1}{T}\int_0^T W_{\Delta I}(\nu, t)\mathrm{d}t \simeq \frac{1}{\pi T}\int_0^T\int_0^\infty |R_{EE^*}(t,\tau)|^2 \cos(2\pi\nu\tau)\mathrm{d}\tau\mathrm{d}t \\ & \simeq \frac{1}{\alpha T}\frac{1}{(2\pi)^2\sqrt{\pi}}\frac{\nu_{eff}}{\nu^2}\exp\left(-\frac{\pi^2\nu^2}{\nu_{eff}^2}\right) \end{aligned} \tag{2}$$

Here $|R_{EE^*}(t,\tau)|$ is the current absolute value of the correlation function of complex emission field fluctuations at the observational point, $T = (1/\alpha)(\Delta F/f_o)$ – averaging time of spectrograph, $\nu_{eff} = |\overrightarrow{V}_{eff}|/\ell_{E_H}$ – effective frequency of emission oscillations, $\ell_{E_H} = \ell_0/\sqrt{\bar{s}_H^2}$ (ℓ_0 is an outer scale of screen turbulence, and $\bar{s}_H^2$ is mean square of phase fluctuations at the initial frequency f_o of spectrograph), $|\overrightarrow{V}_{eff}| = |\overrightarrow{V}_H + (z_2/(z_1+z_2))\overrightarrow{V}_s|$ is an absolute value of effective velocity of irregularities, $\overrightarrow{V}_s = (\mathrm{d}\overrightarrow{\rho}_s/\mathrm{d}f)(\mathrm{d}f/\mathrm{d}t)$ – "velocity" of virtual point source effectively observed by spectrograph while sweeping (sweep rate $\mathrm{d}f/\mathrm{d}t$) original stationary wide band source with frequency (spatial) distribution described by $\mathrm{d}\overrightarrow{\rho}_s/\mathrm{d}f$.

It should be mentioned that in case of receiving at constant frequency (say, f_H) the correspondent expression for spectrum of emission intensity fluctuations can be written as

$$W_{\Delta I_c}(\nu) \simeq \frac{1}{\sqrt{\pi}\nu_E} \exp\left(-\frac{\pi^2\nu^2}{\nu_E^2}\right) \tag{3}$$

where $\nu_E = |\overrightarrow{V}_H|/\ell_{E_H}$ is the effective oscillation frequency in the received emission spectrum.

It is also necessary to note that expressions (2),(3) are valid in case of small angle scattering of radio waves on the phase screen when $|z_1 + z_2| \gg |\overrightarrow{\rho}_s|$.

Comparing (2) and (3) one can find that in case of negligible difference between effective and actual velocities of screen inhomogeneities ($|\overrightarrow{V}_H| \gg (z_2/(z_1+z_2))|\overrightarrow{V}_s|$) the following frequency dependence for measured spectra is valid

$$\widetilde{W}_{\Delta I}(\nu) \sim \left(\frac{\nu_E}{\nu}\right)^2 W_{\Delta I_c}(\nu) \tag{4}$$

It follows from (4) that the high frequency ($\nu > \nu_E$) components of the averaged frequency spectrum of intensity fluctuations of the emission received by spectrograph should be notably decreased compared to the fluctuation spectrum at constant frequency.

Moreover, as it follows from (2), in case of $|\overrightarrow{V}_{eff}| \to 0$ (that is possible if $|\overrightarrow{V}_H| \approx (z_2/(z_1+z_2))|\overrightarrow{V}_s|$ and $\overrightarrow{V}_H \uparrow\downarrow \overrightarrow{V}_s$) fluctuations of the emission intensity received by the spectrograph should have an abnormally narrow spectrum. Generally, an essential narrowing of the fluctuation spectrum in a wide frequency band should be observed with spectrographic investigations of wide band distributed sources compared to the case of quasi–monochromatic signal receiving. Taking into account that $\overrightarrow{V}_s \sim \mathrm{d}\overrightarrow{\rho}_s/\mathrm{d}f$ (see above) the effect of dramatic narrowing of the fluctuation spectrum can be used to estimate the characteristic spatial scale of a distributed wide band source if actual drift velocity of screen irregularities $\overrightarrow{V}_H$, distance from the screen to the source z_1, and to the receiver z_2, and sweep rate of spectrograph $\mathrm{d}f/\mathrm{d}t$ are known.

As a rule, sweep rate of spectrographs is not enough high (for instance, spectrograph of WIND spacecraft has $\mathrm{d}f/\mathrm{d}t \approx 0.75$ MHz/s [Bougeret et al., 1995]). In this case one can estimate that the effective velocity of a virtual source "moving" along Jupiter's L–shell (for a distributed Io-A type source $L \approx 6$ [Lyons and Williams, 1984]) $V_{eff} \approx (z_2/(z_1+z_2))|\overrightarrow{V}_s|$ is essentially less than drift velocity of Solar wind plasma irregularities

$|\vec{V}_H|$ (spatial parameter $|d\vec{\rho}_s/df|$ for an Io-A type source at frequencies of $f = 3$–10 MHz is $|d\vec{\rho}_s/df| = 200$ km/MHz. Distances from the screen can be estimated as $z_1 \approx 3.8$ AU and $z_2 \approx 0.2$ AU, so $V_{eff} \approx 7.5$ km/s while $|\vec{V}_H| \approx 300$–400 km/s [Lyons and Williams 1984]).

At the same time comparable values of $|\vec{V}_H|$ and V_{eff} can be reached with observations by "fast" decametric spectrographs, for example, at Nançay ($df/dt \approx 50$ MHz/s [Lecacheux et al., 1998a]). Hence, there exists a potential possibility to observe essential narrowing of intensity fluctuation spectrum of Jupiter's HF emission received by a spectrograph in a wide frequency band. Spatial scale of a distributed wide band radio source can be estimated based on this effect.

3 Frequency–temporal cross correlation of Jovian DAM emission fluctuations

In the previous section measurements of frequency (space) averaged characteristics of Jupiter's decametric radio emission sources were discussed. At the same time, measurements of statistical characteristics of received radio emission allow more precise spectrographic investigations of Jupiter's DAM source spatial structure.

Following Gershman et al. [1984], and using the technique of frequency spectra of radio emission intensity fluctuations calculations described above, it is easy to show that normalized frequency–temporal correlation function of emission intensity fluctuations obtained from spectrographic records with frequency separation of $\delta f_m = m \cdot \delta f$ and temporal separation of $\tau_m = m \cdot \tau$ ($m = 1, 2, 3, \ldots$) in the interesting case of refractive scattering of short wave radio emission from the distributed source can be written as [see also Alimov et al., 1997]:

$$\Gamma_{\Delta I}(\delta_m, \tau_m) \simeq \frac{1}{1 + (\delta_m D)^2} \exp\left\{ -\frac{[\vec{V}_H \tau_m - \Delta\vec{\rho_s}(\delta_m)]^2}{\ell_E^2} \right\} \tag{5}$$

Here $\delta_m = \Delta f_m / 2f_0$ is the relative frequency shift of the spectrograph (f_0 is a central frequency of the spectrograph band), $\delta_m \ll 1$; $D = 2L/k_0\ell_E^2$ is a wave parameter ($D > 1$), $\ell_E = \ell_0/\sqrt{\bar{s_0^2}}$ – characteristic scale of complex field of emission ($\bar{s_0^2}$ – mean square radio wave phase fluctuations at frequency f_0), $\Delta\vec{\rho}_s(\delta_m) \approx (z_2/(z_1 + z_2))\vec{\rho}_s(\delta_m)$, $\vec{\rho}_s(\delta_m)$ is the characteristic spatial source separation in the aperture plane, with simultaneous frequency separation Δf_m around central frequency f_0. Expression (5) was derived supposing stationarity of the radio wave scattering process in time and frequency inside a narrow frequency band ΔF around the central frequency f_0 ($\Delta f_m \ll \Delta F \ll f_0$). The band ΔF should be chosen to satisfy the stationarity condition of the studied process of short radio wave scattering off interplanetary plasma irregularities inside ΔF. At HF it can be estimated as $\Delta F \approx 1$–3 MHz [see Gershman et al., 1984; Alimov and Lapidus, 1973].

It is followed from (5) that the temporal shift of the correlation maximum of the receiving emission intensity fluctuations relative to the origin can be, in principle, used to determine the local value of $\vec{\rho}_s(\delta_m)$ if the Solar wind inhomogeneities drift velocity $\vec{V}_H$ is known for a given geometry of the experiment. It should be kept in mind that the accurateness of such measurements is determined by the temporal (τ) and frequency (Δf) resolution of the spectrograph. The accuracy is higher the lower those parameters are. High precision measurements of the spatial separation of characteristic scales of emission sources can be optimized using continuous many–frequency observations. This technique was applied to Jupiter's decametric radio emission observations in a two–frequency version [Maeda, 1992]. Measurement of a set of two–frequency cross correlation functions in a wide frequency band allows in principle the reconstruction of a detailed spatial distribution of Jupiter's DAM emission sources.

However, it is necessary to keep the following in mind: Strictly speaking, $\Delta\vec{\rho}_s(\delta_m)$ in expression (5) could characterize the effective spatial separation of emission sources separated in frequency by Δf_m instead of the actual one. The point is that, as it is known [Gershman et al., 1984], temporal shift (relative to $t = 0$) of the frequency cross correlation maximum of received emission intensity fluctuations can be caused also by radio wave refraction. In our case it can be radio wave refraction in Jovian atmosphere directly close to the emission sources. Then, taking into account [Gershman et al., 1984], $\Delta\vec{\rho}_s(\delta_m)$ in expression (5) can be written as

$$\Delta\vec{\rho}_s(\delta_m) = \Delta\vec{\rho}^{\,a}_s(\delta_m) + \Delta\vec{\rho}_{refr}(\delta_m) \approx \Delta\vec{\rho}^{\,a}_s(\delta_m) + 2\delta_m\vec{\theta}_0 z_L \tag{6}$$

Here $\vec{\theta}_0 \approx (4\cdot 10^7/f_0^2)\partial N_m/\partial\vec{\rho}$ is the refraction angle of a radio wave with the frequency f_0 in the Jovian atmosphere, $\partial N_m/\partial\vec{\rho} = \int_0^{z_\ell} \mathrm{d}z\partial N(\vec{\rho}, z)/\partial\vec{\rho}$ is the full transverse gradient of electron density $N(\vec{\rho}, z)$ in a planetary plasma layer with the effective thickness z_L, $\Delta\vec{\rho}^{\,a}_s(\delta_m) = (z_2/(z_1+z_2))\vec{\rho}_s(\delta_m)$ ($\vec{\rho}_s(\delta_m)$ is an actual value of the spatial separation between separate sources with a frequency separation Δf_m in the aperture plane of the distributed emission source).

It can be supposed that fluctuations of electron density gradients are small while averaged on relatively long observational intervals ($\langle\partial N_m/\partial\vec{\rho}_s\rangle \approx 0$), and $\langle\Delta\vec{\rho}_s(\delta_m)\rangle \approx \Delta\vec{\rho}^{\,a}_s(\delta_m)$. However, separate realizations can have comparable values of $\Delta\vec{\rho}^{\,a}_s(\delta_m)$ and $\Delta\vec{\rho}_{refr}(\delta_m)$, and there could be a considerable error in the determination of $\Delta\vec{\rho}^{\,a}_s(\delta_m)$.

Under experimental conditions of two frequency ($f_1 = 22.2$ MHz, $f_2 = 21.87$ MHz, $\delta_m \simeq 8\cdot 10^{-3}$) observations of an Io-A source on September 13, 1988, [Maeda, 1992] with Solar elongation of $\varepsilon \approx 105°$ and characteristic values of layer thickness $z_L \approx 0.8\,R_J$ (R_J is Jupiter's radius) and Solar wind velocity $|\vec{V}_{sw}| \approx 300$–$400$ km/s, values of $|\Delta\vec{\rho}_s| \approx 0$ can be observed if there were electron density gradients of about $|\partial N_m/\partial\vec{\rho}| \approx 10^5$ cm^{-3} in the source region, although averaged over the whole observational interval (about one hour) the electron density gradient was about zero $|\partial N_m/\partial\vec{\rho}| \approx 0$.

The actual value of $|\vec{\rho}_s|$ can be estimated as $|\vec{\rho}_s| \approx 180$ km. Such an estimation can be obtained taking into account experimental conditions [Maeda, 1992] for elementary Io-A sources frequency separated by $\Delta f_m \approx 330$ kHz around a central frequency $f_0 \approx 22$ MHz from the results of mean delay measurements of maximum frequency–temporal cross correlation function $\langle\tau\rangle \approx 0.06$ s, and most probable values of $|\vec{V}_H| \approx |\vec{V}_{sw}| \approx 300$–$400$ km/s and $z_1 \approx 3.8$ AU and $z_2 \approx 0.2$ AU (see relation (5) and Maeda [1992]).

However, above estimations of the full transverse gradient of electron density $|\partial N_m/\partial \overrightarrow{\rho}| \approx 10^5$ cm^{-3} obviously seem to be too high for Jupiter's magnetosphere [Physics of the Jovian Magnetosphere, 1983]. Therefore, it is hardly possible to explain large variations in time delays for maximum values of frequency–temporal cross correlation function observed in experiments [Maeda, 1992] by short wave refraction in Jupiter's atmosphere.

There is one more possibility to change the value of spatial separation of elementary emission sources $\Delta\overrightarrow{\rho}_s(\delta_m)$ (see relation (5)). It can be caused by appropriate variations of magnetic field strength in the Jovian magnetosphere. Actually, taking into account the cyclotron nature of Jupiter's decametric emission, the frequency which is determined as $f(\text{MHz}) = f_B \approx 2.82\, B(\text{Gauss})$; then one can approximate $\Delta\overrightarrow{\rho}_s(\delta_m)$ as

$$\Delta\overrightarrow{\rho}_s(\delta_m) \approx \left.\frac{\partial\overrightarrow{\rho}_s}{\partial B}\right|_{\overrightarrow{B}} \Delta B(\delta_m), \tag{7}$$

where $\overrightarrow{B}$ and ΔB are the mean magnetic field and its strength deviation in the emission generation region for sources with Δf_m frequency separation.

As a first approximation, it is possible to consider $\Delta B(\delta_m)$ to be a constant while $\partial\overrightarrow{\rho}_s(\delta_m)/\partial B|_{\overrightarrow{B}}$ can be considered as a constant only at short temporal intervals up to few ten seconds, and it is varying around its mean value $\langle\partial\overrightarrow{\rho}_s(\delta_m)/\partial B\rangle$ with a time scale of several minutes. Then, it is evidently followed from (7) that relative variations of $\overrightarrow{\rho}_s(\delta_m)$ and $\partial\overrightarrow{\rho}_s(\delta_m)/\partial B$ are equal. Relative variations of $|\overrightarrow{\rho}_s(\delta_m)|$ were in the order of unity in experiments [Maeda, 1992], because variations of the measured values of delays of frequency–temporal cross correlation function maximums in the received emission were similar to mean values of those delays (compare (5) and [Maeda, 1992]). In this case the mean square value of the magnetic field variations $\sigma_{\nabla B} = \langle\partial B/\partial\overrightarrow{\rho}_s\rangle$ in the emission source region can be estimated from (7) and

$$\Delta f_m \simeq 2.82\Delta B(\delta_m) \tag{8}$$

In the case of the experiments analyzed above [Maeda, 1992] $\Delta f_m \approx 0.33$ MHz, and from relation (8) one can obtain $\Delta B \approx 0.12$ Gauss. Taking into account that $(z_2/(z_1 + z_2))\langle|\overrightarrow{\rho}_s|\rangle = |\overrightarrow{V}_H|\langle\tau\rangle$ and estimations for $z_1 \approx 3.8$ AU, $z_2 \approx 0.2$ AU, $|\overrightarrow{V}_H| \approx 150$ km/s, and $\langle\tau\rangle \approx 0.06$ s it can be found from (7) that $\langle|\overrightarrow{\rho}_s|\rangle \approx 180$ km and, correspondingly, $\sigma_{\nabla B} \approx \Delta B(\delta_m)/\langle|\overrightarrow{\rho}_s|\rangle \approx 7\cdot 10^{-4}$ Gauss/km.

So, strong variations of measured values of the delays of frequency–temporal Jupiter's DAM fluctuations cross correlation function maximums obtained in [Maeda, 1992] could be explained by relatively slow (several minutes) and strong (relative value about unity) variations of weak spatial magnetic field gradients in the planetary atmosphere.

Therefore, measurements of the frequency–temporal cross correlation of received decametric emission intensity fluctuations can be used as an effective diagnostic tool to detect weak spatial gradients of magnetic field in planetary atmospheres, in particular, in the Jovian magnetosphere.

4 Conclusion

Spectrographic investigations of statistical characteristics of received radio emission, in principle, can be used to determine structure of distributed sources of Jupiter's decametric emission. Measurement of the spectrum of emission intensity fluctuations by wide band (few ten MHz) spectrograph provides information on the averaged spatial structure of the studied radio source. On the other hand, measurement of the frequency–temporal cross correlation of emission fluctuations using fast sweeping in a relatively narrow frequency band (few MHz) of receiver provides information on the fine structure of the distributed source.

Results of theoretical calculations were applied to the results of known experimental investigations of Jupiter's DAM emission. In particular, it was shown that measurements of the frequency–temporal cross correlation of received decametric radio emission intensity fluctuations can be used as an effective diagnostic tool to detect week spatial magnetic field gradients in the Jovian magnetosphere.

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